The Tradable Pollution Permit Exercise: Three Additional Tools

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Abstract

The trading of permits has been a significant innovation in controlling pollution in the past two decades. Ando and Ramirez Harrington (2006) developed an informative classroom exercise demonstrating the effectiveness of a tradable permits market. We provide three additional pedagogical tools for the exercise that will further engage students. First we show how students with an intermediate microeconomics background can algebraically and graphically calculate the market clearing permit price. Second, for advanced students we show how the cost minimizing allocation of pollution control is achieved using a Lagrangian equation and explain the economic interpretation of the shadow price. Third, we show how to solve the Lagrangian first order conditions computationally using Excel's matrix inverse tool for each firm's emissions reductions and the shadow price.

Keywords: instructional game, tradable discharge permit, horizontal summing, Lagrangian shadow price

JEL Codes: A22, A23

Motivation

The trading of pollution permits has been one of the significant innovations in controlling pollution in the past two decades. Ando and Ramirez Harrington (2006) developed a highly informative classroom exercise for showing students how effective the tradable pollution permit market is. Students are put into groups that represent six firms, and each firm is given a marginal abatement cost curve in equation form. Initially, each firm is asked to calculate the amount that

will be abated and what the abatement cost will be under a uniform standard. A tradable permit system is then introduced, with the instructor acting as a Walrasian auctioneer. By tabulating the firms' demands for permits at various prices, the instructor can show how the market clears at a particular price. Firm and industry level total abatement, cost of abatement, and costs net of revenue from permit sales can be tabulated.

While this exercise is very effective, a few simple enhancements make it even more engaging and useful as a pedagogical tool, particularly for more advanced students. First, as it stands the equilibrium price is essentially determined by trial and error. This is an appropriate approach for explaining the process to students with limited backgrounds in Economics. But students who have completed intermediate microeconomics can be shown how to calculate the equilibrium market clearing tradable permit price algebraically and shown the equilibrium graphically. Second, for the more advanced students (e.g. Master's students who have taken Mathematical Economics) we show how the cost minimizing allocation of pollution control is achieved using a Lagrangian equation and explain the economic interpretation of the shadow price (which is equal to market clearing price). Third, we also show the students how to solve the Lagrangian first order conditions computationally using Microsoft Excel's matrix inverse tool to calculate the amount of emissions reductions for each firm and the value of the shadow price.

Calculating Equilibrium Permit Prices

The firm's marginal abatement cost (MAC) curve is essentially the demand curve for pollution permits. At lower permit prices, the firm's net demand for permits will increase, implying that their demand for emitting pollutants will increase, and vice versa. Having explained this concept, instructors can demonstrate how to sum the firms' (MAC) curves

horizontally in order to derive the industry-level MAC_{total} curve, which is essentially the industry demand for pollution. Students will recognize the horizontal summation concept from their intermediate microeconomics course text (e.g, Pindyck and Rubinfeld (2013, p. 128)). This technique is useful in other applications (e.g., summing individuals' demand curves in order to create a market demand curve), so it may be worthwhile for instructors to review this method. Having completed this exercise we can equate the MAC_{total} curve to the perfectly inelastic supply of permits to calculate the equilibrium price, both algebraically and graphically.

Using the particular example created by Ando and Ramirez Harrington (2006), we start with the MAC curves of the six firms:

Firm 1: $MAC(E) = 4,000 - 2E$

Firm 2: $MAC(E) = 8,000 - 4E$

Firm 3: $MAC(E) = 10,000 - 5E$

Firm 4: $MAC(E) = 4,000 - E$

Firm 5: $MAC(E) = 8,000 - 2E$

Firm 6: $MAC(E) = 10,000 - 2.5E$

It is necessary to invert the curves to horizontally sum the curves. Also it is important to tell the students to recognize that since there are three unique vertical intercepts for the six curves there will be two kink points on the horizontally summed MAC_{total} three section curve. The inverse MAC curves are:

Firm 1: $E = 2,000 - (1/2)MAC$

Firm 2: $E = 2,000 - (1/4)MAC$

Firm 3: $E = 2,000 - (1/5) MAC$

Firm 4: E = 4,000 - MAC

Firm 5: E = 4,000 - (1/2)MAC

Firm 6: E = 4,000 - (2/5)MAC

The MAC_{total} curve will have the following three sections:

Section One of MAC_{total}:

For MAC values below $4,000 all six curves are summed yielding:

$$E = 18,000 - (57/20) MAC_{total}$$

Inverting this curve yields

$$MAC_{total} = \$6,315.79 - (20/57)E$$

Section Two of MAC_{total}:

For MAC values starting at $4,000 to just below $8,000, only curves 2, 3, 5 and 6 are summed

yielding:

$$E = 12,000 - (27/20)MAC_{total}$$

Inverting this curve, yields

$$MAC_{total} = \$8,888.89 - (20/27)E$$

Section Three of MAC_{total}:

For MAC values starting at $8,000 and above, only curves 3 and 6 are summed yielding:

$$E = 6,000 - (3/5) MAC$$

Inverting this curve, yields

$$MAC_{total} = \$10,000 - (5/3)E$$

Thus the kink points on the MAC_{total} curve occur at MAC = $8,000 and E = 1,200; and at

MAC = $4,000 and E = 6,600. It will be helpful to graph the individual firms' inverted demand

curves as well as the horizontal summation. We find an Excel spreadsheet to be a useful addition

to this exercise. This permits a simple demonstration of the horizontal summing process. An

example of such a graph can be found in Figure 1. (The Excel spreadsheet used to generate this graph is available from the authors on request.)

The Ando and Ramirez Harrington (2006) exercise uses a fixed vertical supply of permits of 8,880 tons. This supply curve can be added to the graph. Students will immediately notice that the intersection of supply and demand occurs in Section One in the range of MAC values below $4,000 and it becomes straightforward to determine the equilibrium price of permits by inserting the value of $E = 8,880$ in $MAC_{total} = \$6,315.79 - (20/27)E = 6,315.79 - (20/57)8,880 = \$3,200$. Thus the market clearing equilibrium price is $3,200 per ton of pollution permit, which the students will notice is the same as was discovered in the *tatonnément* process described in Ando and Ramirez Harrington (2006). Should instructors wish, they can select a different quantity of tradable permits. If the amount selected falls between 1,200 and 6,600, it is easy to show on the graph that the second section of the inverted MAC_{total} curve above should be used (that is, $MAC_{total} = \$8,888.89 - (20/27)E$). Quantities below 1,200 will involve the third section of inverted MAC_{total} curve.

Ando and Ramirez Harrington (2006) ask their students to predict the equilibrium price before the game begins. After the game is over, the above algebraic and graphical illustration can be used to show how the equilibrium price can be derived. Alternatively, this can be assigned as homework (for students with the appropriate background). Cartwright and Stepanova (2012) note that there are significant benefits from requiring students to write a report about the experiment. Students could be asked to solve this problem algebraically and then diagram it in Excel.

Teaching Pollution Control: A Cost-Minimization Approach

The classroom discussion of this game can also be expanded for the benefit of more advanced students (e.g. Master's students who have taken Mathematical Economics). It is not difficult to show that the cost minimizing allocation of pollution control can be derived by means of a Lagrangian equation. Having seen this, students may more readily grasp the economic interpretation of the shadow price (which is equal to market clearing price).

In an unregulated market the six firms are emitting collectively 18,000 tons of pollution. If the governmental authority decides to allow only 8,880 tons of emissions, this implies that 9,120 tons of emissions must be reduced. The goal is achieve the 9,120 tons reduction at the lowest possible cost. Note that we are now modeling the problem from the perspective of controlling pollution rather than modeling the amount of pollution emitted. This means that the MAC curves are increasing functions of pollution control, q_i. Specifically this means that

Firm 1: $MAC(q_1) = 2q_1$

Firm 2: $MAC(q_2) = 4q_2$

Firm 3: $MAC(q_3) = 5q_3$

Firm 4: $MAC(q_4) = q_4$

Firm 5: $MAC(q_5) = 2q_5$

Firm 6: $MAC(q_6) = 2.5q_6$

This can be modeled using a Lagrangian equation.

1) $\mathcal{L} = \sum_{i=1}^{6} \int_{q_i=0}^{q_i^*} MAC_i \ dq_i + \lambda[\bar{Q} - \sum_{i=1}^{6} q_i]$

The interpretation of this equation is that integral, i.e. the area under each firm's MAC curve from zero units to the optimal amount of emissions (E) controlled, q_i^*, is the total abatement cost for the firm. These costs are summed for the six firms. The constraint states that $\bar{Q}$ units, e.g. 9,120 tons, of pollution must be reduced. λ is the Lagrange multiplier. (See Chiang and Wainwright (2005, p. 350) for details of the Lagrange multiplier method.) The first order conditions are:

$$2a) \ \frac{\partial \mathcal{L}}{\partial q_1} = MAC_1 - \lambda = 0 \Longrightarrow 2q_1 = \lambda$$

$$2b) \ \frac{\partial \mathcal{L}}{\partial q_2} = MAC_2 - \lambda = 0 \Longrightarrow 4q_2 = \lambda$$

$$2c) \ \frac{\partial \mathcal{L}}{\partial q_3} = MAC_3 - \lambda = 0 \Longrightarrow 5q_3 = \lambda$$

$$2d) \ \frac{\partial \mathcal{L}}{\partial q_4} = MAC_4 - \lambda = 0 \Longrightarrow q_4 = \lambda$$

$$2e) \ \frac{\partial \mathcal{L}}{\partial q_5} = MAC_5 - \lambda = 0 \Longrightarrow 2q_5 = \lambda$$

$$2f) \ \frac{\partial \mathcal{L}}{\partial q_6} = MAC_6 - \lambda = 0 \Longrightarrow 2.5q_6 = \lambda$$

$$2g) \ 9{,}120 - q_1 - q_2 - q_3 - q_4 - q_5 - q_6 = 0$$

These seven equations can be expressed in matrix form as:

$$
A \qquad\qquad x \;=\; b
$$

$$
\begin{bmatrix}
2 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 4 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 5 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 2 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 2.5 & -1 \\
1 & 1 & 1 & 1 & 1 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ \lambda
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 9{,}120
\end{bmatrix}
$$

The solution for the vector *x,* which contains the socially optimal amount of pollution control for each firm as well as the shadow price, can be solved as

$x = A^{-1}\, b$

The students can be shown how to solve this set of equations in Excel. The matrix *A* can be inverted using the Excel command MINVERSE command. The first step requires writing the *A* matrix in Excel, which is a 7x7 matrix. Next, one should select a 7x7 area in the spreadsheet (preferably nearby the A matrix) and begin typing =MINVERSE (give the range of cells for the *A* matrix) and hit CtrlShiftEnter simultaneously. The A^{-1} matrix will fill the selected 7x7 area and will look like this:

0.41228	-0.04386	-0.03509	-0.17544	-0.08772	-0.07018	0.17544
-0.04386	0.22807	-0.01754	-0.08772	-0.04386	-0.03509	0.08772
-0.03509	-0.01754	0.18596	-0.07018	-0.03509	-0.02807	0.07018
-0.17544	-0.08772	-0.07018	0.64912	-0.17544	-0.14035	0.35088
-0.08772	-0.04386	-0.03509	-0.17544	0.41228	-0.07018	0.17544
-0.07018	-0.03509	-0.02807	-0.14035	-0.07018	0.34386	0.14035
-0.17544	-0.08772	-0.07018	-0.35088	-0.17544	-0.14035	0.35088

The A^{-1} matrix can be premultiplied to the b vector by using the =MMULT command. A 7x1 area needs to be selected for the results area of the x vector. Then type =MMULT(array1, array2) and hit CtrlShiftEnter simultaneously. Array1 is the A^{-1} matrix and array2 is the b vector. The results for vector x will look like this:

1600

800

640

3200

1600

1280

3200

Thus we have determined the social cost minimizing pollution control for each firm. The first six values are the optimal pollution control amounts for the Firms 1 through 6, respectively. The seventh value is λ. Firm 1 should control 1,600 tons (which implies emissions of 400), Firm 2 should control 800 (emissions of 1,200), Firm 3 should control 640 (emissions of 1,360), Firm 4 should control 3,200 (emissions of 800), Firm 5 should control 1,600 (emissions of 2,400) and Firm 6 should control 1,280 (emissions of 2,720). The students will recognize these values are the same as the values for tons abated in Table 4 of Ando and Ramirez Harrington (2006). Thus we have shown that the amounts of pollution that each firm voluntarily chooses to control (after market for permits has cleared) are the socially optimal (i.e., the cost minimizing) amounts. This is a powerful result.

The last value in the *x* vector is the value for λ, the Langrange multiplier, which is $3,200. This is the shadow price of controlling another unit of pollution. The value of the Lagrange multiplier constitutes a measure of the effect of a change in the constraint via the parameter $(\bar{Q})$ on the optimal value of the objective function (Chiang and Wainwright (2005), p. 454.) At the optimal allocation of pollution control across the six firms, an increase in pollution control from 9,120 to 9,121 would raise social cost by $3,200. This is the essence of a shadow price that a constraint entails.

Of course, the first order conditions of the Lagrangian equation can be used to demonstrate the concept that values of MAC for all firms should all be equal, at the optimal pollution control levels. This can be shown to be true because the MAC curve values must all be equal to λ simultaneously. This MAC equality condition is a result that is usually explained intuitively or graphically using two curves (Tietenberg and Lewis, 2012, p. 374), which is an appropriate analysis for a student with a limited math or economics background. But for more advanced students, the Lagrangian results will provide some additional enlightenment.

Conclusion

The original version of this Ando and Ramirez Harrington (2006) experiment was designed for students with limited math and economics backgrounds. Our enhancements are intended for a more advanced class. If an instructor teaches a more advanced class or, say, a "piggyback" class of senior undergraduates and Master's students, there is a need for some more in-depth analysis of the problem. We provide three additional pedagogical tools for enhancing the tradable permit game. First, we show how to algebraically and graphically horizontally sum the MAC curves and equate the summed curve to the vertical supply of permits to determine the equilibrium price. Second, we show how to use a Lagrangian equation to determine the optimal

allocation of pollution control across six firms with differing MAC curves. This Lagrangian problem will help the more advanced students utilize some of their skills in an interesting and practical way. Third we show how to use Microsoft Excel to computationally solve for the optimal pollution control amounts for each firm and for the shadow price. Every opportunity to utilize Excel in class should be availed since Excel is such a highly valued tool that all Economics majors should acquire.

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Figure 1: Marginal Abatement Cost (MAC) Curves

Section Three:
$MAC_{total} = \$10{,}000 - (5/3)E$

8,880 tons of emissions

Section Two:
$MAC_{total} = \$8{,}889 - (20/27)E$

Permit Price

Section One:
$MAC_{total} = \$6{,}316 - (20/57)E$

MAC/ton

Emissions (E) (tons)

MAC1   MAC2   MAC3   MAC4   MAC5   MAC6   Price   MACtotal