PERSISTENT MULTIPLE PRICES FOR OSCILLATING DEMAND

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Abstract
The case of oscillating demand has yielded an optimal single preset price and capacity (Anderson, 1974; Sherman & Visscher, 1977). Here under oscillating demand we show when multiple prices will be set in advance for essentially the same service, under monopoly or competition, and how such prices may persist even when consumers are perfectly informed about prices. Pricing in hotels, theatres, sports arenas, airplanes, and other services are in part explained by this argument.

I. Introduction
The demands for many public services (such as gas or electricity) depend in part on alternative conditions that arise randomly (e.g. the weather), and such demands may therefore oscillate randomly between different levels. If a single price and operating capacity for a service must be selected prior to observing the level of demand in a given period, it has been shown that the welfare optimum price and capacity will clear the market at one demand level and not the other; see Anderson (1974) and Sherman & Visscher (1977). Here we show that a single price solution is not always the profit maximizing solution; more than one price may be offered by either a monopolist or a competitive industry in this situation, and both (or more generally all) demand levels will be served. The different prices are still set in advance, but they apply to different units of capacity so when demand is low and some units are not filled some prices may not be relevant.

Prices and capacity are set before demand can be observed whenever changing those variables in response to changing market conditions is prohibitively costly; see Carlton (1978). Capacity is difficult to change largely because its construction takes time, and once it is built it cannot costlessly be dissolved. Prices are maintained even in the face of changing demand because it is costly.
to inform customers of a price change. Frequently altered prices may inhibit trade by causing planning difficulties for customers; a stable price might even be a part of an “implicit contract” between seller and buyer that fosters a continuing relationship. The seller is not limited to offering a single preset price, however, even though price and capacity are set in advance. Indeed, our purpose here is to show that in the face of a demand level that oscillates randomly, both a profit-maximizing monopolist and perfectly competitive industry will prosper by assigning different prices to different portions of the total available capacity. Then the solution need not be tailored only to one demand level, but instead may better handle conditions in all possible states of the world.

Persistent differences in prices within the same market are usually explained by consumer search costs (Salop, 1977) or by appeal to an especially lasting disequilibrium situation. Our explanation is based upon neither of these typical arguments. We claim that the price differences will exist in equilibrium even when all consumers know which firms offer which prices. Differences in prices exist in one market commonly when the commodity produced is non-storable. Examples include transportation services, hotels and motels, car-rental services, theatres and movie houses, sports arenas, etc. When multiple prices are set in advance for such services some special distinctions often can be attached to different service units. In commercial transportation a wide range of service classes exists, for example, based on a variety of considerations such as the roominess of the passenger area or the earliness of advanced booking that is required. Hotel and motel rooms come in many different sizes and features, as do rental cars. Theatres and arenas distinguish seats mainly by their nearness to the event of interest, but also by roominess of seating and differences in backrests and cushions. While all of these distinctions are genuine enough, we shall argue here that such distinctions might have to be invented if they were not already there, because suppliers will be able to use them for a purpose quite separate from the quality differences they intrinsically represent. To emphasize such other purposes we shall assume distinctions can be made without apparent cost, and they will be considered only as they facilitate the offering of different preset prices for different units of the service.

We shall examine initially the case of a monopolist facing oscillating demand when customers arrive in the order of their willingness to pay for the service. In this situation, distinguishing separate service qualities is motivated in order to get different prices for them. The higher priced units are sold only when demand is high so capacity for those units is less intensively utilized. When we go on to consider competitive market organization we find these price distinctions will still be warranted. Costs again are actually higher in providing the higher priced service even with constant average capacity cost because the capacity to produce the service will be less intensively utilized, and cost differences warrant price differences at the equilibrium.
II. Price-Setting Monopoly with Arrivals in Order of Willingness to Pay

Consider a monopolist facing oscillating demand and having to set prices before demand is known. In general, when the monopolist can differentiate the units it sells, in order to rationalize different prices for different units it will prefer to charge multiple prices. A multiple preset price strategy will be shown able to maximize profit even when arrivals come in an order that is least favorable to that conclusion, namely when those who place the greatest value on the service are served first. Although he would prefer a higher price across the board when high demand appears, the monopolist cannot forecast with certainty which periods will turn out to have high demands. The next best alternative is to reserve some portion of the market capacity for sale at a higher price, to gain at least some added benefit from high demand even though as a consequence more of that capacity will go unused in slack periods. Since units of output having the same physical characteristics will not in general have the same price in a multiple-price strategy, some non-price rationing mechanism must determine which persons will purchase at which prices.\(^1\) The mechanism is not unimportant to the monopolist.\(^2\) If the monopolist knows, for example, that individuals with lowest willingness to pay are likely to queue up first for service at the low price, he would surely prefer a top price for the last units offered, because the persons who end up buying at the high price are those who value it more and will not be as easily dissuaded by price. This effect of the order in which consumers are served will be shown more rigorously in section III. Here we show that even in the case of the rationing mechanism least favorable to the multiprice hypothesis; i.e., when those with the highest willingness to pay are served first, multiple prices can be motivated.

Consider a simplified environment in which one of two possible demand schedules will be realized next period. The boldness of this assumption is justified on the grounds of tractability, its previous usefulness (Anderson, 1974), and the high likelihood that conclusions here would hold in a generalized model. The situation is illustrated in Fig. 1. Demand \(x_1(p)\) appears with probability \(q\), and \(x_2(p)\) appears with probability \(1 - q\). The rental cost of a unit of productive capacity \(z\) is \(\beta\). There is no operating cost.\(^3\) If \(q = 1\), and demand

\(^1\) For discussion of ordering assumptions, see Visscher (1973).
\(^2\) The monopolist who can set in advance only a single price does not care which consumers are served when demand is high and capacity is scarce; see Sherman & Visscher (1978). When more than one price can be set in advance, however, the monopolist will want to know more about which consumers might be served, since he can collect different amounts from different consumers.
\(^3\) Operating cost can be added without altering the conclusions we draw. We modify the partial equilibrium model of Oliver Williamson (1966) by eliminating operating cost and by having demand either at one level or another in any period, based on chance. Brown & Johnson (1969) extended the Williamson model to deal with uncertain demand but in their model all who receive service in any period pay the same price.
$x_1(p)$ is therefore certain, the most profitable point of operation for the monopolist would be at point $a$, where the profit margin as a percentage of price is equal to the inverse of the demand elasticity. If $q$ is sufficiently small, however, it pays for the monopolist to respond to the possibility of $x_2(p)$ with a price that is higher and capacity that is greater. If $x_2(p)$ is likely, a price is chosen nearer that at $b$, the most profitable if $x_2(p)$ were certain. Such a price is $p^*$ which trades off the expected cost of deviating from the price at $a$ or $b$, given the probabilities that $x_1(p)$ and $x_2(p)$ will appear. The best choice of capacity is then either $z_1$ if $p_1$ is optimal or a larger capacity like $z^*$ (that clears the $x_2(p)$ market) if a price like $p^*$ is optimal. The optimum capacity must clear the market for one of the two possible demand curves. For if $(1 - q)p^* > \beta$ then at $z_1$ the expected marginal revenue from added capacity exceeds the marginal cost and (because $\beta$ is constant) therefore warrants added capacity up to $z^*$ (at which point there is no value to additional capacity; see Sherman & Visscher, 1977).

Assume $z^*$ rather than $z_1$ is the optimum capacity given a best single price $p^*$. By definition of the point $b$, profit would increase by raising price above $p^*$ if the larger demand curve were certain:

$$\left. \frac{dEI}{dp} \right|_{p=p^*} = (p^* - \beta) x_2' + x_2 > 0. \quad (2.1)$$
Now consider subdividing the capacity \( x_2(p^*) \) into two segments, \( \tilde{z}_1 = x_1(p^*) \) and \( x_2(p^*) - \tilde{z}_1 \). Suppose the price \( p^* \) is maintained on the \( \tilde{z}_1 \) units, but a slightly higher price is assigned to the remaining units. Expected profit on the units \( x_2 - \tilde{z}_1 \) is

\[
E\Pi_2 = (1-q)(p-\beta)(x_2-\tilde{z}_1),
\]

and an increase in price above \( p^* \) on those units will increase \( E\Pi_2 \) if

\[
\frac{dE\Pi_2}{dp} \bigg|_{p=p^*} = (1-q)[(p^* - \beta)x_2 + x_2(p^*) - \tilde{z}_1] > 0. \tag{2.2}
\]

Because \((p^* - \beta)x_2 + x_2\) is positive from (2.1), (2.2) can be positive if \( \tilde{z}_1 \) is not large, and that would mean a second price above \( p^* \) for units beyond \( \tilde{z}_1 \) would increase total profit. We know \( p^* \) is determined by maximizing

\[
E\Pi = p[qx_1 + (1-q)x_2] - \beta x_2,
\]

which requires

\[
\frac{dE\Pi}{dp} \bigg|_{p=p^*} = (p^*[qx_1'(1-q)x_2'] + qx_1 + (1-q)x_2 - \beta x_2 = 0 \tag{2.3}
\]

Now we provide an example of demand functions \( x_1 \) and \( x_2 \), and a cost parameter \( \beta \) such that (2.2) and (2.3) are satisfied, and thereby demonstrate that two prices can be most profitable even when those with the highest willingness to pay are served first.

Let \( x_1 = 1-p \) and \( x_2 = A-p \) for \( A > 1 \). Then from (2.3),

\[
p^* = \frac{1}{2}[\beta + q + (1-q)A],
\]

and (2.2) requires

\[
\beta + A - 1 - p^* > 0.
\]

Combinations of \( q \), \( A \), and \( \beta \) that satisfy these two expressions make a second price profitable. Suppose, for example, that \( \beta = 0 \) and \( A = 2 \); then this requirement from (2.2) reduces to

\[
\frac{q}{2} > 0,
\]

demonstrating that a second price is warranted.

We now set out the first order conditions for the two-price, expected profit maximum. Assume that two distinct prices will be set before demand is known. At the profit maximum, the portion of the capacity to which the low price is assigned will just clear the market during low demand. For if the quantity offered at a low price exceeds quantity demanded in low demand periods, \( x_1(p_1) \), then profits in high demand periods are being foregone; a higher price
can be assigned to some of those units because when demand is high a higher price can be obtained. On the other hand, if the quantity offered at the low price is less than \( x_1(p_1) \), then a higher \( p_1 \) could have been chosen with no loss of sales during periods of low demand. Further, as argued above, when those who value the service most are served first, total capacity should just clear the market during high demand, \( z = x_2(p_2) \), if a second price is offered at all. Thus we have expected profits, \( E\Pi \), of

\[
E\Pi = p_1 x_1(p_1) + (1 - q) p_2 [ x_2(p_2) - x_1(p_1) ] - \beta(x_2(p_2)) \quad \text{for} \quad p_2 \geq p_1. \tag{2.4}
\]

The first term represents revenues from sales \( x_1(p_1) \) that are received regardless of which demand schedule appears. The second term is the \textit{additional} expected revenues that are realized with the appearance of demand schedule \( x_2(p) \), an event having probability \( 1 - q \). The final term represents costs of capacity \( z \). Implicit in (2.4) is the assumption that the persons who will be served at price \( p_2 \) are those on the margin of the \( x_2(p) \) schedule. Again, this follows from our assumption that in all periods those with the highest willingness to pay are receiving first the opportunity to purchase at the low \( p_1 \) price.

First order conditions for maximizing \( E\Pi \) by choice of two preset prices \( p_2 > p_1 \), are:

\[
p_1: x_1(p_1) [ p_1 - (1 - q) p_2 ] + x_1(p_1) = 0 \tag{2.5}
\]

\[
p_2: (1 - q) [ p_2 x_2(p_2) + x_2(p_2) - x_1(p_1) ] - \beta x_2(p_2) = 0. \tag{2.6}
\]

From (2.5) we obtain

\[
\frac{p_1 - (1 - q) p_2}{p_1} = \frac{-x_1(p_1)}{p_1 x_1(p)} = \frac{1}{\eta_1} \tag{2.7}
\]

Condition (2.7) states that \( p_1 \) is set to maximize expected profit where period 1 demand elasticity, \( \eta_1 \), is unity only when \( q = 1 \). If \( q < 1 \) price should be higher, because that permits more capacity to be offered at the high price when high demand appears. Note that because \( (1 - q) p_2 \) is the expected opportunity cost of selling a unit at price \( p_1 \), (2.7) is equivalent to the familiar inverse elasticity monopoly pricing rule. From (2.6) we see that (remembering \( \partial x_1(p_1)/\partial p_2 = 0 \)):

\[
(1 - q) \left[ p_2 + \frac{x_2(p_2) - x_1(p_1)}{x_2'(p_2)} \right] = \beta. \tag{2.8}
\]

The term in brackets is the marginal revenue from a price-induced unit increase in capacity if \( x_2(p) \) is certain. Thus, multiplying that term by \( 1 - q \) gives the expected marginal revenue of additional capacity. And added capacity is profitable to the point where expected marginal revenue equals \( \beta \).
Equation (2.8) can be put in the form
\[
\frac{p_2 - \beta/(1 - q)}{p_2} = \frac{1}{\frac{p_2 x_2'(p_2)}{x_2(p_2) - x_1(p_1)}},
\]
If we define the elasticity \( \eta_2 = \frac{p_2 x_2'(p_2)}{x_2(p_2) - x_1(p_1)} \), we have
\[
\frac{p_2 - \beta/(1 - q)}{p_2} = \frac{1}{\eta_2},
\]
a condition that again resembles the familiar monopoly rule for the price-cost margin. A single price might still be more profitable if \( p_2 \) from (2.9) would be equal to or less than \( p_1 \) from (2.7). Nevertheless, we have already shown cases where \( p_2 > p_1 \), so the possibility of two prices clearly exists. Of course \( q \) may be so large it is unprofitable to cater at all to high demand possibilities. In that case only a single price would be charged, excess demand being preferable in the few cases when \( x_2(p) \) appears. Otherwise a two-price strategy will be optimal, even with efficient non-price rationing.

III. Market Competition and Multiple Prices

We have seen that when a monopolist must set prices and capacity in advance, different prices may be charged for the same service in order to profit from the knowledge that in some periods the monopolist will have an unexpectedly large demand for its service. This explains why some firms suspected of exercising monopoly power might make seemingly arbitrary distinctions among units of the commodity they sell.

But persistent multiple prices seem to exist as well in markets not known for the presence of market power. Car rental agencies, for example, exhibit competitive behavior in many dimensions but charge different rates for apparently similar services even though the rates are known in advance and even widely advertised. It might be argued, of course, that subtle quality differences such as speed of service or ease of credit finance justify the price differences. We shall demonstrate that multiple prices could result from competitive behavior even without genuine quality distinctions, however. Price differences have been shown to exist under competition by Prescott (1975) and by Carlton (1978) when consumers were identical.

We show, using the same oscillating demand of the previous section, that multiple prices for the same service are possible in competition even when customers differ by willingness to pay and know the price charged by each firm. The multiple competitive prices are not due to late arrivals having a willingness to pay systematically greater than those who make early reservations. The source of the result is that in periods of high demand all firms could
have successfully charged a price higher than that which clears the market on average. It is only because firms cannot predict demand *ex ante* that they end up choosing in advance a price lower or higher than that which could be sustained if demand could be flawlessly forecasted. The following example shows an incentive for firms in competitive markets to use this fact by offering some service at a higher price and expecting lower utilization of capacity.

The point of this example is to show that different prices may result in a competitive market if demand oscillates, even when the quality differences among units are artificial. Some portion of the total market capacity is assigned a preset price higher than that assigned to the rest of capacity. These multiple prices are maintained in periods of both low and high demand but presumably the low-price units will be filled first. In low demand periods the high price units will go unused. In high demand periods the low-price units continue to be filled at the “bargain” price, even though, *ex post*, higher prices could have been achieved.

Assume that firms are risk neutral and face the market demand situation described in Fig. 2. Demand $x_1(p)$ exists at any point in time with probability $q$, and $x_2(p)$ is present whenever $x_1(p)$ is not. The cost of a unit of production capacity, $z$, is $\beta$. Marginal variable cost of production is zero.
A proof that competition can result in multiple prices in this market is straightforward. Assume all firms choose a single price $p^*$. We know that at equilibrium $p^* > \beta$, for otherwise all firms would lose money. And the strict inequality, $p^* > \beta$ cannot persist. Because a quantity demanded of at least $x_1(p^*)$ is certain every period, $p^* > \beta$ can earn positive profits for some firms each period and competitive price cutting will drive price to $p^* = \beta$. Entry will occur until market capacity is $x_1(\beta)$.

The question now is whether the market can support still greater capacity if demand is greater $(1 - q)$ of the time. Additional capacity priced at $p = \beta$ will lose money because it will be filled only a fraction $1 - q$ of the time. But additional capacity is profitable if the price received when that capacity is used is

$$p \geq \frac{\beta}{1 - q},$$

(3.1)

for then the expected revenues of $(1 - q)p$ will cover costs $\beta$ on that additional unit of capacity.

Whether the reservation price of the marginal user in the high demand periods will satisfy (3.1) clearly depends upon

(i) the probability $1 - q$,
(ii) the vertical separation of $x_2(p)$ viz. a viz. $x_1(p_1)$, and
(iii) the order in which demand is served when excess demand appears.

From (3.1) we see that multiple prices are more likely the smaller is $q$ (i.e., the more likely is $x_2(p)$) and the greater is $x_2(p)$; each tends to make greater the expected reservation price of the marginal user in times of high demand, ceteris paribus. The order in which consumers are served also will influence the likelihood that multiple prices will arise, as we shall now show.

If customers who are willing to pay the most for a service are the first ones served, for example, then when demand is high no consumer will be willing to pay for service beyond $x_1(\beta)$ more than

$$P_H = x_2^{-1}(x_1(\beta)).$$

On the other hand, if from among those willing to pay at least the going price consumers who value the service least are served first (Visscher, 1973), the highest amount any customer will be willing to pay when demand is high will be

$$P_L = x_2^{-1}(0).$$

The consumer willing to pay the most when demand is high will not yet be served in the latter case, and will still be in the market. Obviously $P_L > P_H$, meaning that if consumers who value the service least are served first a second
price can be higher. Thus a second price is more likely to be found in the market when consumers who value the service least are served first. Although \( p_H \) will be higher under monopoly market organization than under competition it will still be less than \( p_L = x_2^{-1}(0) \). So the same conclusion that multiple prices are more likely when consumers who value the service least are served first will hold also under monopoly organization, as asserted in section II above.

If multiple prices can be sustained, the competitive market price in periods of low demand is \( \beta \) and sales \( x_1(\beta) \), whereas in periods of high demand the price is \( \beta \) for sales of \( x_1(\beta) \) and \( \beta/(1-q) \) for sales of \( z-x_1(\beta) \). If no multiple price can be sustained, \( p = \beta \) and \( z = x_1(\beta) \). Thus there should always be some capacity priced at marginal cost that is fully utilized whether or not a second price can be sustained.

We can sum up our finding as follows. Persistent multiple prices can arise in this example because if industry capacity just clears the market when \( x_1 \) occurs at a price of \( p^*_L = \beta \), there can remain incentive to enter. For a new seller can set a price above \( p^*_L = \beta \), and although no sales will be made in \( q \) of the periods, up to \( x_2(p_L) - x_1(p_L) \) can be sold at the higher price \( p_L \) in \( (1-q) \) of the periods. These additional units of capacity can earn the competitive rate of return if the reservation price of the marginal user on demand curve \( x_2 \) equals or exceeds \( \beta/(1-q) \). Thus risk-neutral entrants will continue to augment industry capacity until \( p_L = \beta/(1-q) \) is observed in the market.

This analysis can be generalized to more than two demand levels. In each case \( \beta \) divided by the appropriate probability of occurrence at each demand range will place a lower limit on the required price. For judging the minimum price to be expected for service in a demand segment, the appropriate probability of occurrence is the cumulative probability for all demands up through and beyond that particular level. With this adjustment the analysis presented here for the case of two levels of demand becomes a model for analysis of any number of demand levels.

We might emphasize that we assume no supply response is possible within any one time period; output must be chosen on a longer run basis. But we have assumed an ordering that would make multiple prices least attractive, an ordering where those who value the service most have priority. If reservations are used to accomplish such an ordering, consumers will have either to pay for their reservations, or to incur a penalty for cancellation if reservations are not used. If they could cancel reservations without penalty, consumers would always reserve space at the lower price on the chance that they might want to use it and thus assure themselves an opportunity for service up to capacity at the lowest price. Rationing under the reservation system would then be the same as under a first-come, first-served rule. Of course, whether a reservations policy assures service for those with the highest willingness to pay or, rather, for others whose plans are least subject to change, is an open question.
IV. Conclusion

When demand for a non-storable service oscillates among different levels due to a common influence on consumer behavior, like the weather, what is essentially the same service may be found selling at different prices. This situation can persist even if consumers are perfectly informed at any one time about prices, and indeed the persistence of multiple prices requires that consumers know of the price differences. Multiple prices depend on suppliers being imperfectly informed about demand in any one period, and so being motivated to announce in advance higher prices for capacity that will not always be used. The existence of multiple prices is not due to monopoly market organization, but will result also under competition in the same circumstances.

References


